

Resonant tunneling of acoustic waves through a double barrier consisting of two phononic crystals

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Abstract. – We present the acoustic analog of resonant tunneling through a double barrier in quantum mechanics. Pairs of identical phononic crystals, in both 2D and 3D, were assembled and separated by a uniform spacer, forming a resonant cavity. The ultrasonic transmission exhibits resonant peaks at frequencies inside the band gaps, where ultrasound tunneling through each phononic crystal occurs. On resonance, the measured group time is large and even predicted to increase exponentially with the thickness of the crystals in the absence of absorption, while off resonance very fast speeds are found.

Recently, intriguing phenomena in quantum mechanics, such as the Aharonov-Bohm effect [1], weak localization in disordered media [2] and maybe one of the most striking, tunneling [3], have been revisited using acoustic or elastic waves. In these studies, one of the great advantages lies in the macroscopic character of the samples under study. Another is the possibility of directly measuring the phase of the wave field. Thus acoustic waves seem to be particularly appropriate for studying dramatic effects affecting the velocities that can be defined for any wave field (phase, group, transport velocities). For example, by performing group velocity measurements, Yang *et al.* have recently shown that inside a phononic band gap crystal acoustic tunneling is involved [3, 4]. Phononic band gap crystals are periodic structures, analogous to photonic crystals for electromagnetic waves, that forbid propagation of acoustic waves for a certain range of frequencies. Thus, in the band gap, the waves are evanescent, and are similar in character to the solutions of the Schrödinger equation for a particle of energy E inside a barrier of height V_0 when $V_0 > E$. Yang *et al.* found that, inside

a phononic band gap, the group velocity increases linearly with sample thickness, a rather remarkable effect that is a signature of tunneling in quantum mechanics [5].

Here we use phononic crystals to explore the acoustic analog of another quantum phenomenon, that of resonant tunneling through a double potential barrier, and investigate both its stationary and dynamic behavior. Resonant tunneling is intriguing, not only because the transmission coefficient through the double barrier is predicted to be unity at resonance even though the transmission probability through a single barrier may be extremely small, but also because the group time can be shown to increase exponentially with barrier thickness. Can similar phenomena be observed for classical waves? To answer this question, we have measured both the transmission coefficient and the group velocities through pairs of 2D or 3D phononic crystals separated by a uniform medium, which forms a resonant cavity. Evidence for resonant tunneling is demonstrated by a large peak in the transmission when the thickness of the cavity approaches a multiple of half the wavelength. Further evidence comes from the behavior of the group velocity. While off resonance we find very fast group velocities that are larger than any of the velocities in the constituent media and are compatible with a tunneling mechanism, at the resonant frequencies, the reverse holds and very low velocities are indeed measured. However, unlike the predictions for the quantum case, their evolution with crystal thickness is found to saturate. This effect is attributed to loss mechanisms, one of which makes the interference responsible for the band gap no longer completely destructive, an effect that has no quantum counterpart.

Our 3D resonant tunneling system consists of a 7.05-mm-thick aluminium slab enclosed between two identical phononic crystals. These crystals are made from two to four layers of a fcc arrangement of tungsten carbide beads in water, a system that possesses a complete phononic band gap near 1 MHz for 0.8-mm-diameter beads [3,4]. The layers are perpendicular to the [111] direction. To facilitate the investigation of the group time dependence on the thickness of the individual crystals, we have also built resonant cavities from two 2D square arrays of steel rods immersed in water, separated by an integral number of lattice constants. The rod diameter is 0.8 mm whereas the lattice constant is 1.5 mm. The volume fraction (22%) is not large enough to observe a complete band gap within a single crystal, but this does not matter here, since evanescent waves are only required in one direction (the [10] direction in these experiments).

The ultrasonic waves transmitted through both 2D and 3D samples were measured by placing them in a water tank between two transducers whose central frequencies were 500 kHz in the 2D case and 1 MHz in the 3D case. The samples were insonified by a short input pulse, which was planar to a good approximation, and the transmitted signals were detected in the far field by the receiving transducer, digitized and downloaded to a computer. The transmitted signals were averaged over different positions of the sample to detect only the spatially coherent wave, *i.e.* the wave that remains after averaging over any disorder or imperfections in the crystals. Figure 1 compares typical signals transmitted through 2D and 3D double crystals with their respective input pulses.

To measure the frequency dependence of the transmitted amplitude through the two resonant cavities, we determined the transmission coefficient from the amplitudes of the transmitted and input signals at each frequency, using a Fourier transform technique [3]. Typical results are presented in fig. 2. In the 3D case shown in fig. 2(a), each phononic crystal contains 3 layers of beads and the cavity thickness was such that a single resonance peak is observed in the band gap. The peak amplitude is large relative to the deep minimum in the transmission due to the band gap, but it is not unity due to absorption, as we explain further below. In 2D, the transmission through two 4-layer crystals separated by a 10.5 mm cavity (corresponding to 6 missing rows) is presented in fig. 2(b). In the stop band, several resonant peaks are

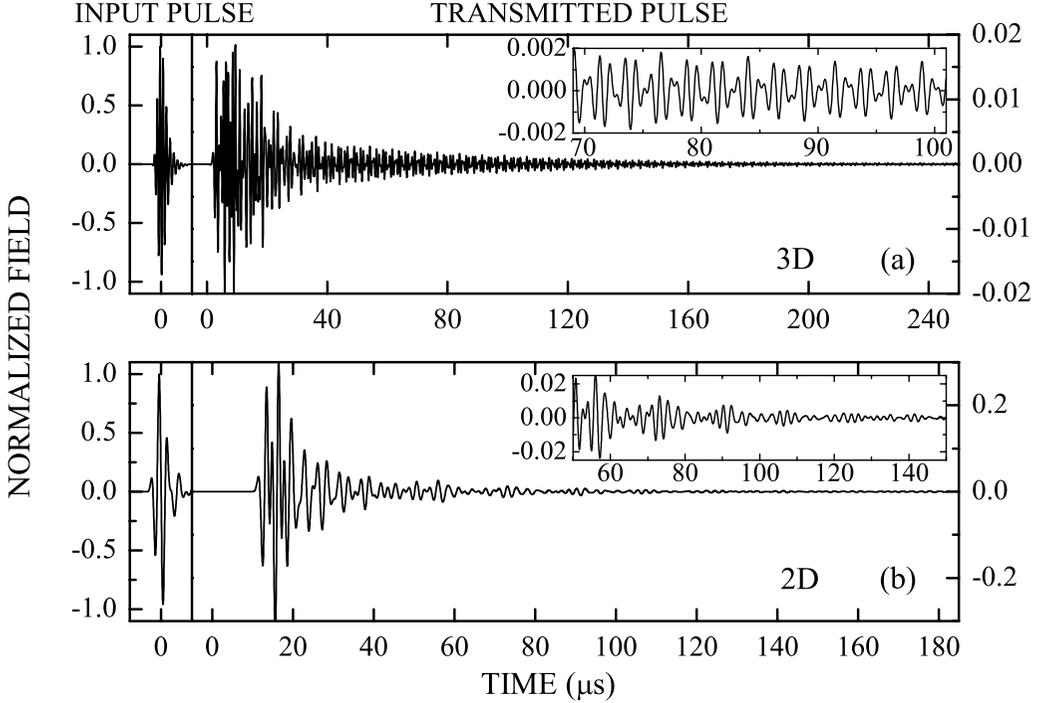


Fig. 1 – Input and transmitted pulses through (a) 3D and (b) 2D double phononic crystals. The number of layers in each crystal is 3 in this 3D example, and 4 in 2D.

observed, which, as shown in fig. 1, are also manifest in the time domain as wave packets separated by the time needed to travel back and forth inside the cavity. In both 2D and 3D, the resonant peaks occur at frequencies that differ slightly from those of a high-reflectivity Fabry-Pérot interferometer: $f_n = nv_p/(2d)$, where v_p is the phase velocity in the cavity, d is the cavity width and n is an integer. For the 3D crystals, the difference is only 4%, while for the 2D system, the difference is somewhat larger. This difference is related to the wave penetration depth in the crystals [6].

Following the procedure described in ref. [4], we also investigated the dynamics of resonant tunneling by measuring the group time t_g and group velocity v_g as a function of frequency inside the gap (see fig. 3). At resonance, the group time is found to be as much as 60 times larger than the time to propagate directly through the cavity (water in the 2D case, aluminium in the 3D case); since the tunneling time is very short [3], this indicates that a long lifetime can be associated with each resonance. By contrast, off resonance we find that the group time is less than the time to propagate through any of the materials from which either the crystals or the cavities are made, due to the very fast group velocity encountered in tunneling. The difference between the group velocities on resonance and just off resonance is striking: depending on the thickness and the evanescent decay length of the phononic crystals, the velocity varies by as much as a factor of 70, encompassing both “supersonic” ($9v_{water}$) and “subsonic” ($v_{water}/8$) sound.

Before quantitatively interpreting these data using appropriate phononic crystal models, we briefly review the predictions of quantum mechanics for the resonant tunneling of a particle through a double rectangular barrier. By applying the usual boundary conditions that the

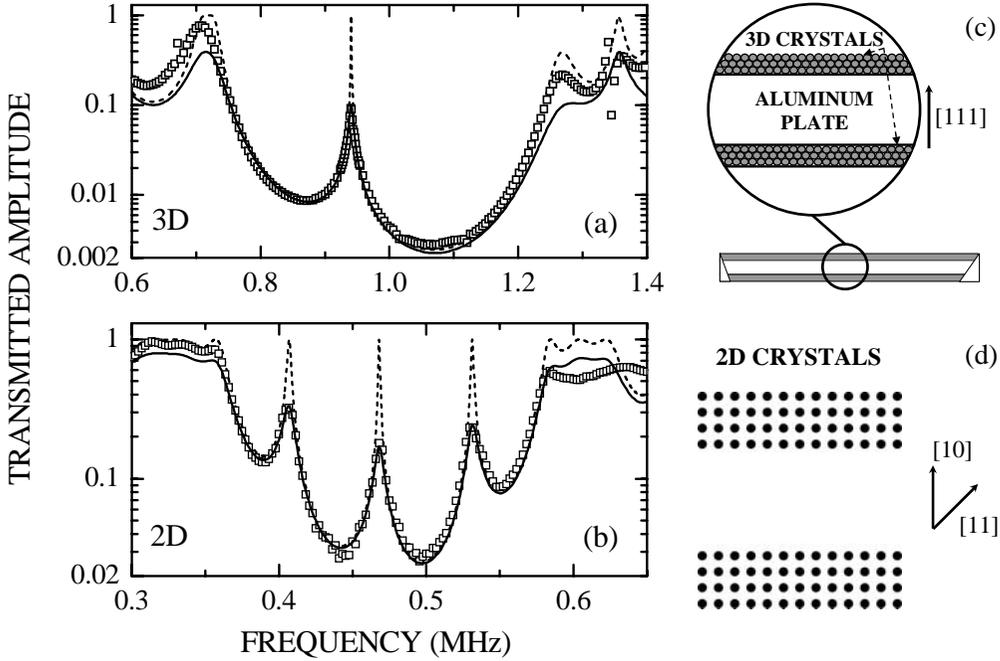


Fig. 2 – Amplitude transmission coefficient through (a) 3D and (b) 2D double phononic crystals. Theoretical predictions with and without absorption (solid and dashed lines, respectively) are compared with the experimental data (symbols). The corresponding crystal structures and crystallographic orientations are shown in (c) and (d).

wave function and its derivative be continuous at the boundaries, the transmission coefficient T when a plane wave is incident on a double barrier can be readily obtained from the more familiar single-barrier result using a transfer matrix technique, giving

$$T = \frac{e^{-2ikL}}{1 + S^2(1 - \varepsilon^2/4) + i\varepsilon CS + S^2(1 + \varepsilon^2/4)e^{2ikd}}. \quad (1)$$

Here $k = \sqrt{2mE/\hbar^2}$ is the wave vector of the incident plane wave, $\kappa = \sqrt{2m(V_0 - E)/\hbar^2}$ is the imaginary wave vector of the evanescent waves in the barrier, $\varepsilon = \kappa/k - k/\kappa$, L is the barrier width, $C = \cosh(\kappa L)$, $S = \sinh(\kappa L)$ and d is the separation between the barriers. When the incident energy E matches one of the energy levels of the potential well formed between the barriers, resonant transmission occurs and $|T| = 1$. For opaque barriers where $\kappa L \gg 1$, the transmission coefficient (1) has maxima when $e^{2ikd} = -[(1 - \varepsilon^2/4) + i\varepsilon]/[1 + \varepsilon^2/4]$, giving the following resonance condition: $kd = \frac{1}{2} \tan^{-1}[\varepsilon/(1 - \varepsilon^2/4)] + (n + \frac{1}{2})\pi$. When the incident energy is half the barrier height, $k = \kappa$ and this condition reduces to $\lambda = 2d/(n + \frac{1}{2})$, which resembles the classical Fabry-Pérot resonance condition apart from the additional $\frac{1}{2}$ in the denominator that arises from the non-negligible barrier penetration depth. By expanding (1) about the resonance in powers of $\Delta k = k - k_{res}$, it is straightforward to show that the resonance becomes exponentially narrow as the width of the barriers increases ($\Delta k \approx (2/d)e^{-2\kappa L}$ when $E = V_0/2$) and the delay time associated with the resonance is exponentially long compared

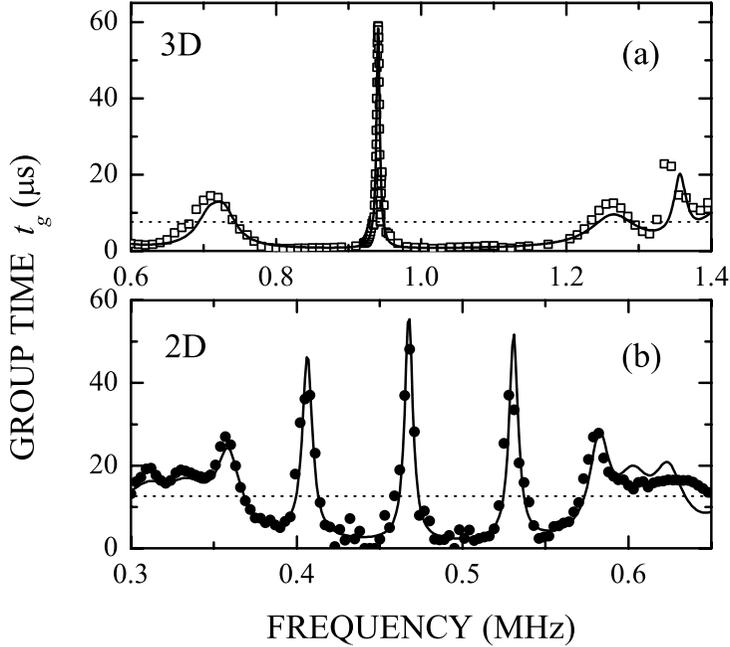


Fig. 3 – Frequency dependence of the measured group time through (a) 3D and (b) 2D double crystals (symbols), compared with the predictions of the MST and 1D model, respectively (solid lines). The dotted lines represent the times to propagate directly through samples in which the phononic crystals are replaced by water (the crystal matrix material).

with the travel time between the barriers: $t_g = d\phi/d\omega = (d\phi/dk)/v_g \approx (d/2v_g)e^{2\kappa L}$, where ϕ is the transmitted phase⁽¹⁾. Thus, the group time is inversely proportional to the tunneling probability through a single barrier.

To demonstrate theoretically the very close analogy that exists between resonant tunneling in quantum mechanics and in phononic crystals, and also to account for the differences that are observed in real phononic materials due to absorption, we have used Multiple Scattering Theory (MST) [7] to calculate the transmission coefficient and the group time for the 3D double phononic crystals. MST is ideally suited for crystals with simple scattering geometries and has been shown to give accurate results for the band structure and transmission through phononic crystals consisting of solid spherical scatterers in a liquid matrix. Figure 2(a) shows the MST predictions for the transmission coefficient, both with and without absorption. When there is no absorption, a narrow peak in the transmission coefficient is predicted at a resonance frequency that is in excellent agreement with experiment⁽²⁾, and the peak value is indeed 1, as in the quantum case. In our phononic crystals, however, there are also losses, which can be included in the MST by allowing the moduli of the constituent materials to be complex, successfully accounting for the reduction in the resonant-peak magnitude that is observed experimentally. The solid curve in fig. 2(a) is a best fit to the data with an absorption coefficient (imaginary wave vector) $\kappa_{abs} = 0.066 \text{ mm}^{-1}$ in the water of the phononic crystal

⁽¹⁾When $E \neq V_0/2$, the prefactors are more complicated, but the exponential dependence on κL still dominates the behaviour of both Δk and t_g .

⁽²⁾Note that there are no adjustable parameters in this calculation, the only parameters being the measured velocities and densities of the constituent materials.

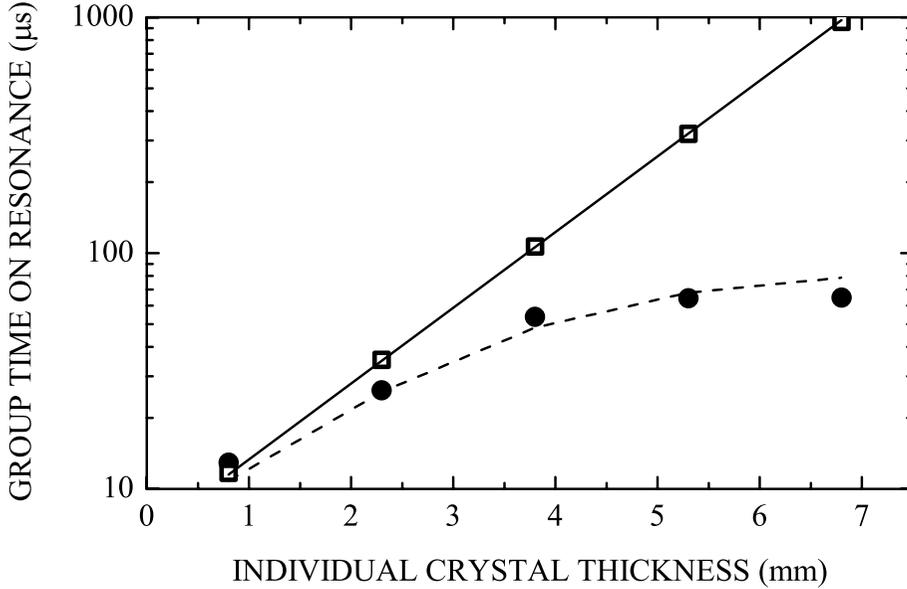


Fig. 4 – Evolution of the group time for the resonance at 468 kHz in the 2D samples *vs.* the thickness of the individual crystals. Experimental results (solid circles) are compared with the 1D model with absorption (dashed line) and without absorption (open squares), and with the predicted exponential dependence $\exp[2\kappa L]$ (solid line), where $\kappa = 0.37 \text{ mm}^{-1}$ was measured from the exponential decrease in transmission through a single crystal. Similar results are found in 3D.

and zero elsewhere. Thus, the dominant loss contribution⁽³⁾ comes from intrinsic absorption in the phononic crystals, which was shown previously to significantly influence the measured tunneling time and to a much lesser extent the transmission through single crystals [3, 4]. The effect of absorption in the phononic crystals on resonant tunneling can be understood in simple physical terms by the so-called two-modes model [3, 4]. In a phononic crystal, absorption cuts off the long multiple-scattering paths so that the destructive interference leading to the band gap becomes incomplete. This incomplete destructive interference leads to a small propagating component, which increases the leakage from the cavity and lowers the quality factor of the resonance, causing the resonant-peak height to be reduced. Losses also affect resonant tunneling in the 2D crystals in the same way; in this case, we capture the basic physics using a simple 1D layered model [8], since we have not yet developed the MST transmission code for 2D. In this 1D model, the thickness of the steel and water layers is adjusted to account for the characteristics of the experimental band gap, and absorption is introduced by allowing the impedance and wave vector of the water layers and cavity

⁽³⁾Cavity losses appear to be relatively unimportant in this case, and were minimized by carefully selecting the material, accurately polishing the spacer flat and parallel, and reducing the angular spread of the input beam. Because of the significant pulse propagation delay encountered on resonance (fig. 3), it is important that the sample-to-transducer distances be sufficiently large that reflections between sample and transducer do not overlap the tail of the directly transmitted pulse through the sample. For the thickest samples, where this condition could no longer be satisfied without complications from edge effects, the transmitted pulse had to be truncated before the signal had died away, leading to a reduction in the measured peak height and group time; however, even in this case, it was still possible to determine the true peak height and group time accurately by measuring these quantities as a function of truncation time and fitting these results with predictions of the MST, thus enabling a reliable extrapolation to the untruncated signal limit to be obtained.

between the crystals to be complex. Excellent agreement is found with experiment when $\kappa_{abs} = 0.011 \text{ mm}^{-1}$ for both the water layers and cavity⁽⁴⁾, as shown in fig. 2(b).

The MST and 1D models, using the absorption coefficients that were found by fitting the transmission data, also give an excellent quantitative description of the frequency dependence of the measured group time (figs. 3(a) and (b)). Furthermore, these models also demonstrate how absorption cuts off the expected exponential increase in the group time with thickness. These predictions are in good agreement with the experimental data, which show an increase in group time with thickness that saturates for the thicker samples (see fig. 4). The physical basis for the observed reduction in group time can again be explained by the two-modes model: since absorption in the phononic crystals leads to a small propagating mode in the band gap, the leakage from the cavity is increased and the dwell time of the pulse on resonance is reduced.

We have presented both theoretical and experimental results for the resonant tunneling of acoustic wave pulses through a pair of phononic crystals separated by a uniform medium, which forms a resonant cavity. These experiments are analogous to the resonant tunneling of a particle through a double barrier, but since they are performed with classical waves, the full wave function can be measured, allowing phase information to be obtained in addition to the stationary and dynamic aspects of resonant tunneling that have been emphasized here. The transmission measurements performed through both 2D and 3D structures show resonant peaks; on resonance, the measured group time becomes very large as a consequence of the long lifetime of the resonances, but the expected exponential increase as a function of phononic crystal thickness is not observed because of loss mechanisms. The data are well explained using Multiple Scattering Theory and a simple 1D model, allowing the non-trivial and somewhat subtle effects of dissipation on the character of evanescent modes in phononic crystals, and on resonant tunneling in particular, to be demonstrated.

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⁽⁴⁾The cavity losses appear to be relatively large for the 2D crystals, possibly because of scattering by defects near the interfaces and diffraction out of the 2D plane.